



Olivetti
Underwood
Programma 10

MATHEMATICS INTRODUCTION



Conventional methods of hand-calculating — using desk calculators and tables — are often impractical for cases in which the work is marginal for a full-scale computer.

Even when a full-scale computer is readily accessible, the hand-calculating involved in checking out the program can require a great deal of time and work.

PROGRAMMA 101 can perform this work in seconds, bringing the full power of a modern computer to the top of every desk. Program routines exist for all of the elementary mathematical functions as well as for any function that can be represented by a power series and most of these are compact enough to be used in more elaborate programs. Thus, formulas involving combinations of elementary functions such as

$$\frac{\sigma}{3} + \tan^{-1} \left[\frac{b \sin (\beta + \frac{1}{3} \sin \alpha c)}{a - c \sin (\beta - \frac{1}{3} \sigma)} \right]$$

can be easily programmed directly at the keyboard and the answers for specific numeric entries will be given in seconds.

Also in the library are routines for solutions of equations, determinants, numerical integration, differential equations, matrix manipulation, and the solution of specific problems in areas such as analytic geometry.

PROGRAMMA 101 will accept a program and constants recorded on a magnetic card as well as directly from the keyboard. It also has an unusual capability that permits it to be operated manually at program stops. The manual instructions are the same as the programmable ones, making possible a flexible and dynamic utilization of the computer.

Programma is an invaluable aid to mathematicians and workers in physical sciences because it saves steps to a large computer for marginal problems and reduces the normal labor of hand-calculating to almost nothing. Because of its elegance and logic, the computer language can be mastered in an hour; the program is always under operator control. Programma requires no complicated control system or compiling, as in time-sharing or Fortran, and a large program library is always on hand to verify or modify the program being used, or to implement second thoughts. All of this enables the operator to use the computer as an immediate extension of his own ideas in his work.

A closer feedback is possible than with any other system. With the Programma, the input-output problem, the time involved in compiling, and similar obstacles tending to separate the man from the computer, are eliminated.

Enclosed in this folder are examples from Programma's mathematical program library. The programs may be used by themselves, in combination with manual operations (e.g., a plot of $ax^2 \sin^{3/2} nx$ could be obtained using only the sine function program), or as routines to be incorporated in more complicated formulas. Many versions of each elementary function are available to suit individual programming requirements, for example, the storage space versus computing time problem ■

ELEMENTARY FUNCTIONS

The Programma mathematical program library uses standard approximations to compute elementary functions to varying degrees of accuracy. Most of these approximations are based on Tchebycheff series with fast convergence. Depending on the nature of the function and the accuracy desired, the series employed may have as many as eight coefficients or as few as three, and, to provide flexibility in using the routine, the constants are in some cases generated in the machine while in others they are placed in registers. The treatment of argument ranges follows similar variation.

The program library contains, in addition to the basic function, some routines of common usage.

Hence for exponentials there is a routine for $\frac{1 - e^{-bx}}{1 - e^{-x}}$

and for Bessel functions there is a routine for $\frac{J_1(x)}{x}$

The programmer may employ the program library as the problem requires and may even generate a power series for other functions in conjunction with the library routine. For this purpose, a number of the routines have been created that are extremely economical in storage requirements. Others are designed for wider variability of the argument or for precision of up to 10 decimal places with a maximum error $< 10^{-9}$ ■



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$$1 - e^{-x}$$

SPECIFICATION:

The argument must be ≤ 14 since $e^{-14} < 10^{-6}$ and the program is limited to six decimal places for fast calculating

FORMULAS USED:

$$1 - e^{-x} = 1 - \frac{1}{e^x}$$

$$e^x = (e^w)^{32}, w = x/32$$

$$e^w = \frac{P(w)}{P(-w)}$$

$$P(w) = 168 + 84w + 18w^2 + 2w^3 + 0.1w^4$$

EXAMPLE:

Given $x = 0.0834$

$$1 - e^{-x} = 0.080017$$

PROGRAMMA PRINTS

0.0834 V
0.080017 A◇

◇ = RESULT



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HYPERBOLIC COSINE

SPECIFICATION:

For 7 or 6 decimal places $0 < x \leq 4$

For 5 decimal places $0 < x \leq 10$

FORMULA USED:

$$\cosh x \approx 1 + \sum_{i=1}^n \frac{x^{2i}}{(2i)!}, \quad n = \frac{n^\circ}{2}, \text{ where}$$

n° depends upon the number of decimal places desired

EXAMPLE:

Given $x = 0.5663$

then for 7 decimal places $n^\circ = 18$
and $\cosh x = 1.1646790$

PROGRAMMA PRINTS

18 D↑

0.5663 V

1.1646790 D◇

◇ = RESULT



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COMMON LOGARITHM

SPECIFICATION:

The argument X must be expressed in normalized form: $X = x \cdot 10^m$, $1 \leq x < 10$
Notice that x is ten times the mantissa used in tables and that m is correspondingly one less than the characteristic for table use.

FORMULAS USED:

$$X = x \cdot 10^m$$

$$\text{Log } X = m + \text{Log } x$$

$$\text{Log } x = C_1W + C_3W^3 + C_5W^5 + C_7W^7$$

$$W = \frac{\sqrt[4]{x} - 1}{\sqrt[4]{x} + 1}$$

EXAMPLE:

Given $X = 483.615$
 $= 4.83615 \cdot 10^2$
Then $m = 2$
 $x = 4.83615$
and $\text{Log } X = 2.6844997$

PROGRAMMA PRINTS

	V
2	S
4.83615	S
2.6844997	c◇

◇ = RESULT



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COMPLEX SQUARE ROOT

SPECIFICATION:

The argument must be introduced by its real part followed by its imaginary part.

FORMULA USED:

$$\sqrt{a + ib} = \pm [s_1 + is_2]$$

$$s_1 = \sqrt{\frac{\hat{M} + a}{2}}$$

$$s_2 = \frac{b}{|b|} \sqrt{\frac{\hat{M} - a}{2}}$$

$$\hat{M} = \sqrt{a^2 + b^2}$$

EXAMPLE:

Given $a = -4.4$
 $ib = i 7.0$

then $s_1 = 1.390685$ $D \diamond$
 $s_2 = 2.516744$ $d \diamond$

PROGRAMMA PRINTS

	V
- 4.4	S
7.0	S
1.390685	D $\diamond$
2.516744	d $\diamond$
- 1.390685	D $\diamond$
- 2.516744	d $\diamond$

$\diamond = \text{RESULT}$



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EXPONENTIAL 10^x

SPECIFICATION:

The argument may be indefinitely large since the result is in normalized form.

That is $10^x = 10^x \cdot 10^m$, where $0 \leq x < 10$

FORMULAS USED:

$$10^x = 10^x \cdot 10^m, 0 \leq x < 10$$

$$10^x = \sqrt{10} (1 + C_1 Z + C_2 Z^2 \dots + C_7 Z^7)^4$$

$$Z = 2^x - 1$$

EXAMPLE:

Given $X = 5.41$

Then $10^X = 10^x \cdot 10^m$
where $10^x = 2.57050381$
 $m = 5.0$

PROGRAMMA PRINTS

5.41	V
5.00000000	A◇
2.57050381	A◇

◇ = RESULT



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SINE, COSINE, AND TANGENT

SPECIFICATION:

The argument may be expressed in either degrees or radians. For sine and cosine $x \leq 90^\circ$. For tangents: $0^\circ \leq x \leq 89^\circ$.

FORMULAS USED:

$$\cos x = \left(2 \cos^2 \frac{x}{2} \right) - 1$$

$$\sin x = \cos \left(\frac{\pi}{2} - x \right)$$

$\cos \frac{x}{2}$ is computed by a series approximation based on Tchebycheff polynomials:

$$\cos \frac{\pi}{2} y \cong \sum_{i=0}^5 C_i T^*(y^2) \quad , \quad T^*(y) = \cos n\theta$$

$$\cos \theta = 2y - 1 \text{ for } -1 \leq y \leq 1 \text{ and } y = \frac{x}{\pi}$$

EXAMPLE:

Given angle $x = 22.^\circ 53200$

$$\sin x = 0.3831993644$$

$$\cos x = 0.9236656578$$

$$\tan x = 0.4148680436$$

PROGRAMMA PRINTS

22.532 V
0.3831993644 b◇

22.532 W
0.9236656578 b◇

22.532 Z
0.4148680436 A◇

◇ = RESULT



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GAMMA FUNCTION

SPECIFICATION:

The argument x must be in the range $0 \leq x < 1$

FORMULAS USED:

$$\Gamma(1 + x) = 1 + C_1x + C_2x^2 + \dots + C_6x^6$$

EXAMPLE:

Given $x = 0.803$

$$\Gamma(1 + x) = 0.932191$$

PROGRAMMA PRINTS

0.803 V
0.932191 A◇

◇ = RESULT

